Fraction Arithmetic

FRAX is a game that is designed to help you and your student/child master fractions, but it does not teach them the basics. I've put together this document to help remind you about basic fraction arithmetic.

Some Terminology

A fraction is a way to describe how much of a whole we are talking about. We are often to used to talking about half of a pizza, or three-quarters of a cup, for example. We write fractions, like one half and three quarters like this:

$$\frac{1}{2} \text{ and } \frac{3}{4}$$

- The **numerator** of the fraction is the number above the bar in the fraction, or the top number. This number describes how many parts of the whole we have left.
- The **denominator** of the fraction is the number below the bar in the fraction, or the bottom number. This number describes how many parts the whole is being divided into.
- A **proper** fraction is a fraction with a smaller numerator than denominator. For example: $\frac{1}{3}$ is a proper fraction.
- An **improper** fraction is a fraction with a bigger numerator than denominator. For example: $\frac{7}{4}$ is an improper fraction.
- A **mixed** fraction is another way of describing an improper fraction. More on this below.

Improper and Mixed Fractions

When we are dealing with improper fractions, often it makes more sense to convert them into mixed fractions. For example, in a recipe, we don't talk about $\frac{7}{4}$ cups of flour, we say $1\frac{3}{4}$ (read 1 and three quarters) cups of flour. It is very easy to turn an improper fraction into a mixed fraction, and a mixed

fraction into an improper one.

To turn an improper fraction into a mixed fraction, like $\frac{7}{4}$ into $1\frac{3}{4}$, you take the top and divide it by the bottom using long division.

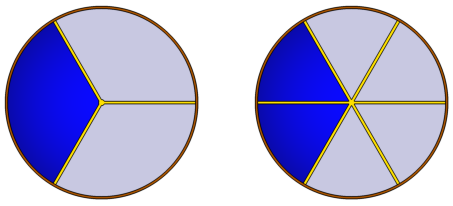
$$\begin{array}{r} 1 \\ 4\overline{)\,7} \\ \underline{4} \\ 3 \end{array}$$

The number on the top, called the **quotient**, is the whole number that goes in the front of the mixed fraction. The number at the very bottom of all of the long division is called the **remainder**, which is the numerator of the fraction part of the mixed fraction. In a mixed fraction, you keep the same denominator as you had with the improper fraction. So in our example, $\frac{7}{4}$ gives 1 as the whole number and 3 as the numerator, from the long division, and so our mixed fraction is $1\frac{3}{4}$.
Another example: $\frac{10}{6}$ becomes $1\frac{4}{6}$ because

$$\begin{array}{r} 1 \\ 6\overline{)\,10} \\ \underline{6} \\ 4 \end{array}$$

Reducing Fractions

When we are describing many parts of a whole something, we can often come up with different descriptions of the same fraction. So for example, if we have the two images:



We can see that the first pizza is divided into 3 parts, with 1 part remaining, and the second pizza is divided into 6 parts, with 2 parts remaining. However, the fractions $\frac{1}{3}$ and $\frac{2}{6}$ represent the same amount of pizza left over! Be able to reduce and compare fractions is an essential skill in estimation.

How to Reduce a Fraction

Reducing fractions involve dividing the numerator and denominator by common factors. A **common factor** of two numbers is a number that divides both of the numbers evenly. For example, if we consider

$$\frac{2}{6}$$

the number 2 divides both the numerator and the denominator. So if we divide 2 by 2 we get 1, and if we divide 6 by 2 we get 3. So we can say that the **reduced form** of $\frac{2}{6}$ is $\frac{1}{3}$, and that these fractions are **equivalent**. If the only number that divides the top and the bottom evenly is 1, then the fraction is already in reduced form.

How to Create Equivalent Fractions

Sometimes we don't actually want a fraction to be in reduced form, such as when we are adding them (described in the next section). If we get a fraction that is already in reduced form, we can take it out of reduced form using the following technique: if you multiply the denominator and the numerator by the same number, you can create an equivalent fraction. For example, if I have the reduced fraction $\frac{1}{4}$, I can multiply the top and the bottom number by 3 to get $\frac{3}{12}$, which is equivalent. When you are doing this, don't forget to multiply the top and the bottom by the same number, its an easy thing to forget!

Adding Fractions

Adding fractions is a lot like regular addition of integers, but there are a couple of extra wrinkles to consider sometimes. The most basic rule to remember, which is a guiding principle for almost all fraction arithmetic, is:

To add fractions you have to make sure the denominators are equal

For example, if we want to add the following fractions $\frac{3}{4}$ and $\frac{2}{4}$, since the denominators are the same, to add fractions you just add the values in the numerators. So

$$\frac{3}{4} + \frac{2}{4} = \frac{5}{4}, \text{ or } 1\frac{1}{4}$$

If the denominators are not the same, we can use the method of creating equivalent fractions to make the denominators the same. For example, if we have $\frac{4}{5}$ and $\frac{2}{3}$ and we want to add them, there is a way to do so that will always work. Multiply the first fraction by the denominator of the second, and the second fraction by the denominator of the first fraction. So, we would do

$$\frac{4}{5} \times \frac{3}{3} \text{ and } \frac{2}{3} \times \frac{5}{5}$$

to get

$$\frac{12}{15} \text{ and } \frac{10}{15}$$

since these denominators are now equal, we can add the numerators:

$$\frac{12}{15} + \frac{10}{15} = \frac{22}{15} \text{ or } 1\frac{7}{15}$$

Multiplying Fractions

Multiplying fractions requires a good deal less work than adding them. The rule to remember here is

Multiply the top times the top, and the bottom times the bottom.

So for example, if we want to multiply $\frac{4}{5}$ and $\frac{2}{8}$, we set up the equation

$$\frac{4}{5} \times \frac{2}{8}$$

which is the same as

$$\frac{4 \times 2}{5 \times 8} = \frac{8}{40}, \text{ or } \frac{1}{5}$$

If you have any questions or comments about anything written on here, please do not hesitate to contact either Andrew or Dan through the FRAX Facebook page.